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Free-rider or contributor: A dilemma in spatial threshold public goods games

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ABSTRACT

Late movers refer to individuals who adjust their strategies after observing the choices of others in a public goods game, can significantly impact the likelihood of public goods provision. This creates a social dilemma: late movers act as free-riders to rely on others to contribute, yet they also risk the public good not being provided if contributions fall short. To address this dilemma, we utilize a spatial threshold public goods game model to examine how the behaviors of late movers influence overall cooperative levels within a population. We propose a probabilistic rule for late movers that captures the dilemma between cooperating to enhance the collective good and free-riding to maximize individual benefits. Our findings indicate that strategic late movers can significantly enhance cooperation by adapting to local cooperation levels. However, an excessively high proportion of late movers may reduce the overall level of cooperation. Additionally, the formation of clusters by late movers results in higher cooperative levels compared to random distribution. Our study provides novel insights into the dynamics of cooperation within the framework of spatial threshold public goods games. By incorporating the role of late movers, we highlight how adaptive strategies can mitigate the free-rider problem and promote cooperative behaviors.

1. Introduction

The exploration of cooperation and its dynamics in social systems has attracted substantial scholarly interest across various disciplines [1–5]. Cooperation, often motivated by mutual benefit, empowers groups to accomplish objectives that are unattainable by individuals acting in isolation. From the formation of early human societies to the complex ecosystems of the present day, cooperative behaviors have been pivotal for the survival of species [6,7], fair distribution of resources [8,9], and social cohesion and stability [10]. For example, by working together, early humans could hunt larger game that would be impossible for an individual to capture alone, which ensures a stable food supply for the group. Additionally, cooperative management of common resources (like water, forests, and fisheries) helps prevent overuse and depletion. Furthermore, regular cooperative interactions build trust and establish norms of reciprocity, which are fundamental for social cohesion and stability.

However, the sustainability of cooperation is not without challenges. One of the most pervasive obstacles is the presence of free-riders—individuals who reap the benefits of cooperation without contributing their fair share [11–13]. These free-riders can undermine the stability and effectiveness of cooperative efforts, leading to the erosion of trust and cooperation over time. Free-riding is particularly problematic in public goods and common resources contexts, where the benefits are non-excludable and non-rivalrous. For instance, in the provision of public safety, basic education, infrastructure, environmental protection, or social welfare, individuals may be tempted to withhold their contributions while still enjoy the benefits. This behavior can lead to under-provision of these goods and depletion of common resources, known as the “tragedy of the commons” [14–16].

The challenge of free-riding necessitates the development of mechanisms and strategies to promote sustained cooperation. One such approach is the establishment of formal institutions and regulatory frameworks that enforce contributions and penalize free-riders [17–19]. Examples include taxation systems to fund public goods, legal regulations to manage common resources, and international agreements to address global issues like climate change. Another approach is the fostering of social norms that encourage cooperative behavior. Social norms, such reciprocity and altruism, can be powerful motivators for individuals to contribute to the collective good [20,21]. Communities that cultivate a strong sense of identity and mutual obligation are often more successful in maintaining high levels of cooperation. Peer monitoring and social sanctions can also play a role in discouraging free-riding and promoting compliance with cooperative norms.

The public goods game provides a powerful tool for exploring the behaviors of free-riders [22–25]. In its basic form, the public goods game simulates a scenario where individuals can choose to either

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contribute to a common pool (cooperate) or withhold their contribution as free-riders (defect). Contributions are then multiplied by a synergy factor and distributed evenly among all participants, irrespective of their individual contributions. This setup mirrors real-world dilemmas where individual interests may conflict with collective goals, as free-riders can obtain higher benefits without contributing.

A critical aspect of this game is the requirement for a certain level of contributions to achieve the collective good. This requirement has been investigated by numerous threshold public goods game models, where the total contributions must exceed a threshold value to produce a public good [26–29]. In this scenario, individuals face a dilemma: they may hope to be free-riders, counting on others to contribute enough to produce the public good, but they also fear that insufficient contributions will prevent the public good from being produced altogether.

To address this dilemma, we delve into the evolutionary dynamics of cooperation within the context of spatial public goods games [30–33]. Here, the spatial structure is represented by a lattice framework where individuals interact primarily with their neighbors. In the study of [34], it is established that conditional cooperators will indeed cooperate when a sufficient number of unconditional cooperators opt to do so. In contrast, our approach introduces the analogous notion of late movers, which permits individuals to adjust their strategies based on the initial decisions made by others. To elucidate the dilemma, we impose a threshold condition for game activation: a public goods game is initiated only if the number of cooperators in the group meets or exceeds a predefined threshold. Specifically, each late mover faces the dilemma to either free-ride or cooperate when the combined count of their own contributions and those of the early movers exceeds the threshold.

Our modeling approach enables us to explore a range of factors influencing cooperation dynamics, including the synergy factor, the fraction of late movers, the game activation threshold, and the spatial distribution of late movers. By systematically varying these parameters, we aim to uncover the nuanced interplay between individual decision-making and the sustainability and robustness of cooperative behaviors.

2. The model

Our model operates within a square lattice framework, characterized by periodic boundaries. This lattice comprises a total of $N = L \times L$ players, denoted as $i \in \{1, 2, \ldots, N\}$. Each player is positioned at a unique grid intersection, surrounded by four adjacent neighbors. At the initial time step $t = 0$, a fraction ρ_L of players is categorized as *late movers*, and the remaining players are designated as *early movers* with the fractions ρ_C and $\rho_D = 1 - \rho_L - \rho_C$ as cooperators and defectors respectively. We assume that each late mover will choose to be a cooperator or defector according to the following rule by Eq. (3).

2.1. The public goods game

At each subsequent time step $t = 1, 2, \ldots$, each player engages in public goods game with their four nearest neighbors, forming a group g comprising $z = 5$ participants (see Fig. 2). The payoff $p_i(g)$ for each player i within the group g is given by the following equation

$$p_i(g) = \begin{cases} \frac{R\sum_{j\in g} s_j}{z}, & s_i = 0, \\ \frac{R\sum_{j\in g} s_j}{z} - 1, & s_i = 1. \end{cases} \tag{1}$$

Here, s_i represents the strategic choice of player i: $s_i = 1$ indicates that player i cooperates, while $s_i = 0$ signifies defection. If player i defects ($s_i = 0$), their payoff is calculated as the total number of cooperators within the group (the total contributions), scaled by the synergy factor $R > 1$, which is distributed among the group members. On the other hand, if player i cooperates ($s_i = 1$), their payoff has an additional cost of 1. This cost represents the personal sacrifice that cooperative players make in order to contribute to the group's overall welfare.

In our spatial public goods game, all public goods games progress concurrently. Each player involves in $Z = 5$ distinct groups — one centered on themselves and the remaining four comprising their nearest neighbors (see Fig. 2). To emphasize, an early mover always has same strategy in different groups (G), whereas a late mover can adapt their strategies based on the specific group. Within this context, the total payoff P_i for each player i can be given as

$$P_i = \sum_{j\in g\in G} p_j(g). \tag{2}$$

2.2. The probabilistic rule for late mover

The strategic choice of each late mover is governed by a probabilistic rule that determines the likelihood of opting for cooperation ($s_i = 1$) in each group of public goods game. Specifically, the probability $P(s_i = 1)$ that each late mover cooperates is defined as

$$P(s_i = 1) = \begin{cases} 1, & \text{if } n_C + n_L = \phi, \\ \exp(-\frac{n_L-(\phi-n_C)}{\phi-n_C}), & \text{if } n_C + n_L > \phi \text{ and } n_C < \phi, \\ 0, & \text{if } n_C \geq \phi \text{ or } n_C + n_L < \phi. \end{cases} \tag{3}$$

Here, n_L represents the number of late movers in a specific group, while n_C signifies the number of cooperators among the early movers. The threshold ϕ is a crucial parameter that determines the minimum number of cooperators required for the game to be *activated* within the group. Specifically, the game will only be activated if the total number of cooperators (including both early and late movers who choose cooperative strategy) exceeds or equals to ϕ. To emphasize, in the absence of game activation within the group, no player will accrue any payoff.

The aforementioned probabilistic rule governing the strategic choices of late movers in public goods games can be further illuminated as follows:

(i) In scenarios where late movers observe that the combined count of early movers who have chosen the cooperation strategy (n_C) and the number of late movers (n_L) precisely matches the threshold value (ϕ) required for the game to be activated (i.e., $n_C + n_L = \phi$), they exhibit a unanimous preference for cooperative strategy. This alignment ensures that the public good will be provisioned, and thus, the probability of any late mover opting for cooperation is maximal, resulting in $P(s_i = 1) = 1$ (see Fig. 1(c)).

(ii) However, in situations where the number of early movers adhering to the cooperation strategy (n_C) falls short of the threshold ϕ (i.e., $n_C < \phi$) but the cumulative total of n_C and n_L exceeds the threshold ϕ (i.e., $n_C + n_L > \phi$), each late mover has the potential to be a free-rider by choosing the defection strategy. This tendency arises from their belief that the contributions from early movers, along with the potential contributions from other late movers, will suffice to meet the minimum threshold (ϕ) for the game to proceed. As free-riders, they can enjoy the benefits of the public good without incurring the costs of contributing. To quantify this probability that late movers choose to be cooperators in such scenarios, we employ an exponential function: $\exp\left(-\frac{n_L-(\phi-n_C)}{\phi-n_C}\right)$. The numerator $n_L - (\phi - n_C)$ captures the gap of which late movers need to fill between its count with the number $\phi - n_C$ of cooperators necessitates the game activation of the group. The larger this gap, the lower the probability of a late mover choosing to cooperate. This reflects the late mover's *mindset of relying on luck* that other late movers will make contributions. Conversely, the denominator $\phi - n_C$ represents the number of additional cooperators required for the game to commence. A higher value of this denominator indicates a greater need for late movers to contribute, thus increasing their probability of cooperation. This reflects the late mover's *mindset of conservatism* (see Fig. 1(b)).

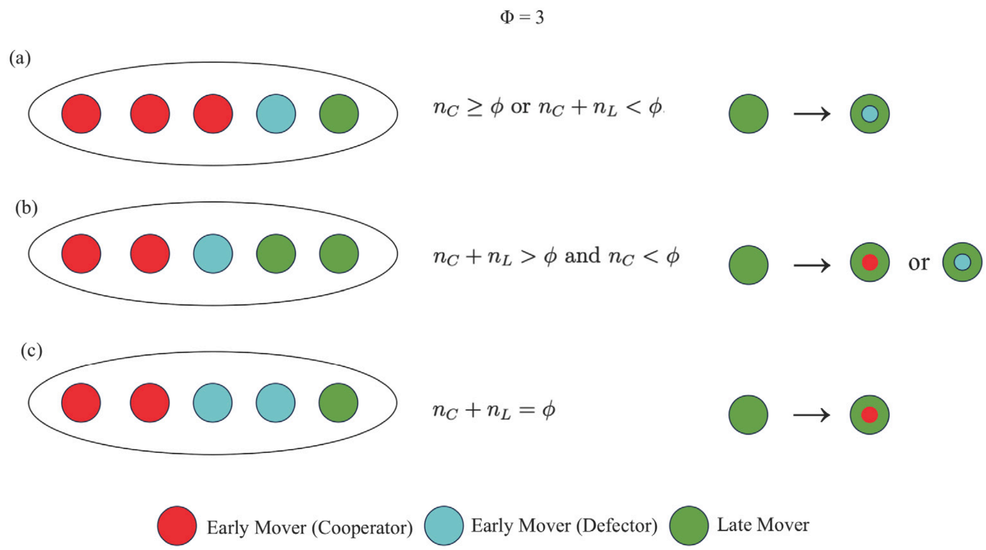

Fig. 1. The illustration of the probabilistic rule of Eq. (3) governing late movers' strategic choices.

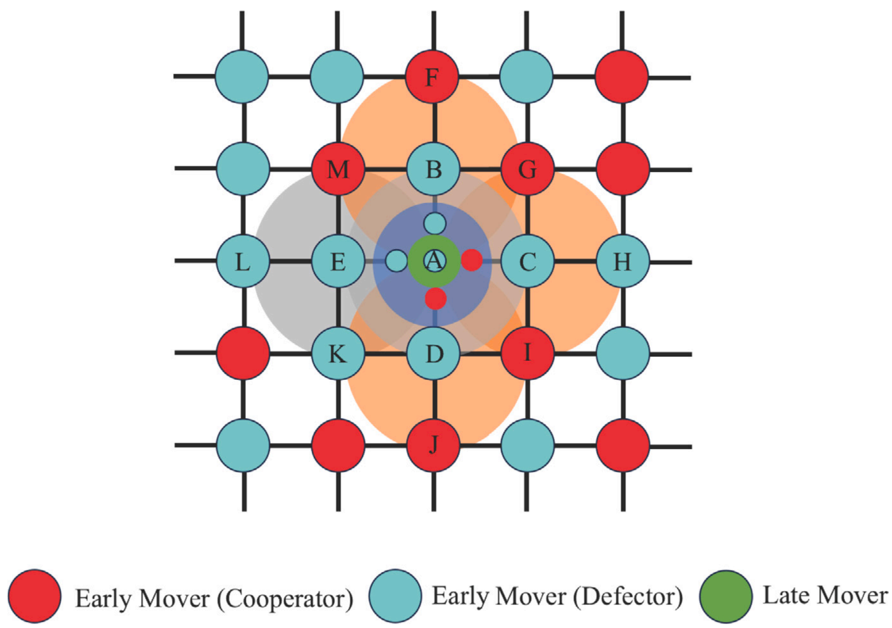

Fig. 2. Snapshot of spatial public goods games centered on a late mover among $Z = 5$ groups with a threshold value $\phi = 3$. In a 5×5 square lattice, cooperators are depicted by red circles and defectors by blue circles. Early movers are initially marked with their respective colors (red for cooperators, blue for defectors), while late movers are shown as green circles encircled by a small blue border. Late movers employ $z = 5$ distinct strategies across all $Z = 5$ groups. Large light gray circles indicate groups that have not yet activated successfully, while large orange circles highlight successfully activated groups. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

(iii) In scenarios where the number of early movers who have chosen the cooperative strategy (n_C) exceeds or is equal to the threshold ϕ required for the game to commence (i.e., $n_C \geq \phi$), late movers exhibit a unanimous preference for adopting a free-riding stance and opting for the defection strategy. This behavior reflects their rational calculation that the contribution from early movers alone is sufficient to reach or surpass the threshold, thereby eliminating the need for their own contributions. As a result, the probability of any late mover choosing to cooperate becomes zero, i.e., $P(s_i = 1) = 0$. Furthermore, late movers demonstrate even greater reluctance to contribute if they determine that even with the cooperation of all late movers, the game would still fail to reach the threshold ϕ. In such cases, where $n_C + n_L < \phi$, late movers perceive the situation as hopeless, as their contribution would not be sufficient to tilt the scales in favor of commencing the game. Therefore, they choose free-riding and opting for the defection strategy (see Fig. 1(a)).

2.3. The strategy learning for early mover

The probability $W(s_j \leftarrow s_i)$ that an early mover i adopts the strategy s_j of a randomly chosen player j (including both early and late movers in the last round of game) from all $Z = 5$ groups is determined by

$$W(s_j \leftarrow s_i) = \frac{1}{1 + \exp(-(P_j - P_i)/\kappa)}, \tag{4}$$

where $\kappa \in [0, \infty)$ is a positive constant that introduces uncertainty in the strategy adoption. A higher κ value implies a greater degree of randomness or noise, meaning that factors beyond just the payoff difference, i.e., $P_j - P_i$, can influence the strategy adoption.

2.4. The cooperative level

In our model, the primary focus is on the fraction of cooperators at each time step t, denoted as $f_C(t)$. This metric captures the cooperative level among the population, providing a snapshot of how cooperation is distributed at any given moment. To understand this metric, it is essential to consider the structure of our model, which includes both early movers and late movers, with each late mover holding multiple strategies.

Each late mover in our model holds $z = 5$ different strategies within all $Z = 5$ groups. This multiplicity of strategies held by late movers necessitates a modification in how we measure the cooperative level. Specifically, the contribution of late movers to the cooperative level should be considered as $Z = 5$ times the number of late movers, reflecting the additional strategic complexity they introduce.

The cooperative level $f_C(t)$ is thus formulated as a ratio of the total number of cooperative actions to the total number of possible actions at each time step t. This is mathematically expressed as:

$$f_C(t) = \frac{N_C(t) + N_{C'}(t)}{N_C(t) + N_D(t) + Z * N_L(t)}, \quad t = \{0, 1, \dots, \infty\}, \tag{5}$$

Here, $N_C(t)$ represents the number of cooperators among the early movers, $N_D(t)$ represents the number of defectors among the early movers, and $N_L(t)$ represents the number of late movers. The term $N_{C'}(t)$ accounts for the number of cooperation strategies held by the late movers, which is calculated as $Z = 5$ times the number of late movers, i.e., $Z * N_L(t)$.

The value of $f_C(t)$ ranges from 0 to 1, where 0 indicates a complete absence of cooperators and 1 signifies a fully cooperative population. This range allows for a straightforward interpretation of the level of cooperation within the population at any given time step.

Additionally, $\overline{f_C}$ refers to the average cooperative level observed in the stable state of the system. This long-term metric provides insights into the enduring patterns of cooperation that emerge from the strategic interactions within the model. By analyzing $\overline{f_C}$, we can understand the equilibrium dynamics of cooperation and how different strategies influence the sustainability of cooperative behaviors over time.

Fig. 2 illustrates the strategic choices of late mover A across all $Z = 5$ groups in a public goods game framework, highlighting which groups become activated. Late mover A opts to defect ($P(s_i = 1) = 0$) in the middle, top, and left groups, $\{A, E, C, B, D\}$, $\{E, L, A, M, K\}$ and $\{B, M, G, F, A\}$, as the combined count of cooperating early movers (n_C) and late movers (n_L) falls below the threshold $\phi = 3$ (i.e., $n_C + n_L = 1 + 1 < 3$ or $n_C + n_L = 0 + 1 < 3$), and the number of cooperators among early movers meets or exceeds the threshold (i.e., $n_C = 3 \geq \phi = 3$). Therefore, the middle and left groups fail to activate. Conversely, late mover A chooses to cooperate ($P(s_i = 1) = 1$) in the bottom and right groups, $\{C, A, H, G, I\}$ and $\{D, K, I, A, J\}$, where $n_C + n_L = 2 + 1 = \phi = 3$, ensuring activation of these groups in the public goods game.

3. Results

3.1. The synergy factor

In Fig. 3(a), we observe a distinct pattern in the evolution of average cooperative levels under varying synergy factors. For lower synergy factor values, such as $R = 1.2$, there is a marked decline in the average cooperative level $\overline{f_C}$. This decline occurs rapidly and stabilizes at a relatively low constant value. This pattern suggests that in environments where the synergy factor is low, individuals are less incentivized to cooperate due to the limited benefits derived from cooperative actions.

Conversely, when the synergy factor is increased to a higher value, such as $R = 5.2$, the behavior changes significantly. In this scenario, the average cooperative level $\overline{f_C}$ shows an initial gradual increase, reflecting a growing inclination among individuals to cooperate. Over time, this tendency stabilizes, and the cooperative level converges to a higher constant value. This observation highlights the critical role of a high synergy factor in promoting cooperation within groups. Specifically, when the benefits of cooperative actions are substantial, individuals are more likely to engage in and sustain cooperative behavior, leading to elevated cooperative levels overall.

3.2. The fraction of late movers

The fraction of late movers (ρ_L) emerges as a crucial factor influencing the evolution of cooperation (see Fig. 3(b)). When ρ_L is relatively low, such as 0.1, the cooperative level tends to gradually decrease. This decline can be attributed to the limited influence of a small number of late movers on the overall system dynamics. In contrast, when the fraction of late movers is higher, such as $\rho_L = 0.5$, there is a notable increase in the cooperative level. This increase indicates that a larger number of late movers significantly boosts cooperation within the system.

However, it is crucial to emphasize that a higher value of ρ_L does not guarantee a perpetually higher cooperative level in the stable state. The relationship between the fraction of late movers and the ultimate cooperative level can be non-monotonic. As Eq. (3) suggests, the ultimate cooperative level is determined by a multifaceted interplay of factors, including the initial fractions of cooperators (ρ_C) and defectors (ρ_D), the fraction of late movers (ρ_L), and the threshold value ϕ.

3.3. The threshold for game activation

The following analysis reveals intriguing insights into how the threshold value ϕ shapes the evolution of cooperation.

In Fig. 3(c), at the extreme lower bound of the threshold, specifically when $\phi = 0$, there is no obstacle for games to activate. This results in late movers consistently choosing defection as free-riders according to Eq. (3). Over time, an initial decline in the cooperative level is observed, followed by stabilization at a constant value.

Conversely, when the threshold value reaches its upper limit, such as $\phi = 5$, a markedly different scenario emerges. In this case, games are activated only when all players choose cooperation strategies. This selective activation ensures that only favorable and productive partnerships are formed, leading to a cooperative level that converges to 1. This indicates that a high threshold value can effectively filter out unproductive interactions and promote cooperation by focusing on partnerships that offer the greatest benefits.

The behavior at intermediate threshold values is particularly intriguing. Generally, higher values of ϕ result in higher levels of cooperation; however, there is a notable exception at $\phi = 1$. Unexpectedly, this specific threshold value leads to a higher ultimate cooperative level compared to slightly higher values, such as $\phi = 2$. This anomaly may be attributed to the probabilistic rule for late movers choosing the cooperation strategy, which is highly sensitive to cases with lower thresholds like $\phi = 1$ and $\phi = 2$.

3.4. The parametric diagrams

Fig. 4 offers an insightful visualization of the intricate interplay between the fraction of late movers (ρ_L) and the synergy factor (R) in determining the ultimate cooperative level within a system. The figure comprises several heatmaps, each portraying distinct scenarios arising from variations in threshold (ϕ) and noise parameter (κ) for the strategy learning.

Starting with the baseline scenario where $\phi = 0$ and $\kappa = 0.1$, we observe that since the public goods game is perpetually activated, late movers tend to be free-riders, resulting in a ultimate cooperative level of zero except for very high value of R (as seen in Fig. 4(a)). However, when the threshold increases to middle values (e.g., $\phi = 1, 2, 3$), we

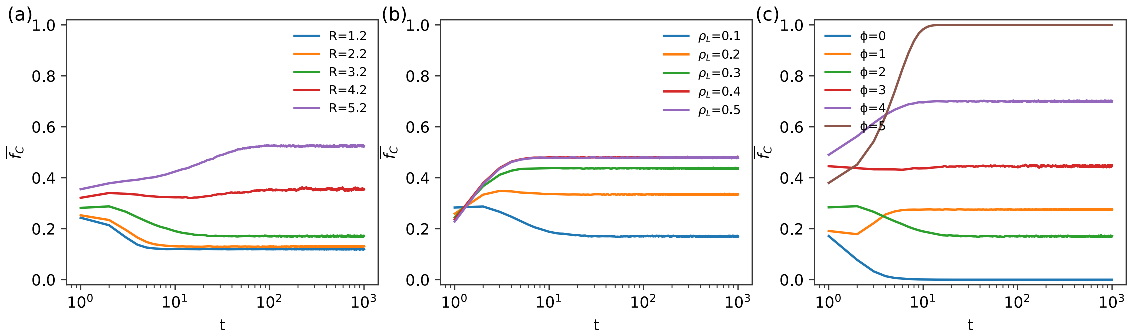

Fig. 3. The evolution of the average cooperative level $\overline{f_C}$ within a $N = 100 \times 100$ square lattice with $\rho_C = \rho_D = (1 - \rho_L)/2$. Settings: (a) $\phi = 2$, $\kappa = 0.5$, $\rho_L = 0.1$; (b) $\phi = 2$, $\kappa = 0.5$, $R = 3.2$; (c) $R = 3.2$, $\kappa = 0.5$, $\rho_L = 0.1$; (d) $\phi = 2$, R=3.2, $\rho_L = 0.1$. Each data point is the average of 50 independent simulations.

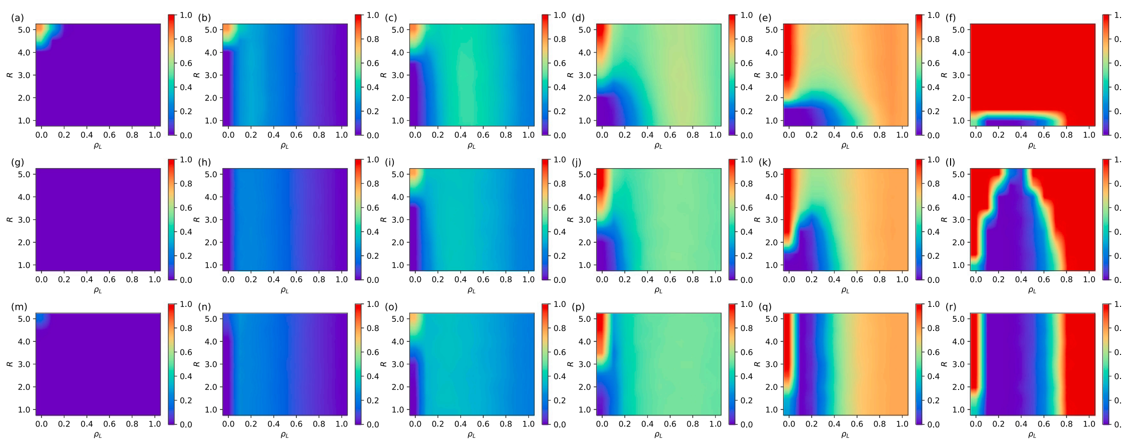

Fig. 4. The ultimate cooperative level in the (ρ_L, R) plane with $\rho_C = \rho_D = (1 - \rho_L)/2$. Left to right panels correspond to $\phi = \{0, 1, 2, 3, 4, 5\}$. Similarly, the panels from top to bottom showcase different κ values: $\kappa = 0.1$, $\kappa = 1$ and $\kappa = 10$.

notice that the relationship between the fraction of late movers (ρ_L) and the ultimate cooperative level is non-monotonic (Fig. 4(b–d)).

A particularly noteworthy scenario emerges when we consider the highest threshold value of $\phi = 5$. At this threshold, under low noise ($\kappa = 0.1$) and higher $R \geq 2$, the system achieves a perfect cooperative level of 1 regardless of the values of R and ρ_L (Fig. 4(f)). However, when noise is increased to a moderate level ($\kappa = 1$), the system landscape changes drastically, with two critical boundary lines emerging (Fig. 4(l)). Below these boundaries, cooperation breaks down completely, resulting in a cooperative level of zero. This shift indicates the fragility of cooperation in the presence of high noise in the strategy learning and a high threshold.

3.5. Special cases

3.5.1. The cluster of late movers

We delve into the effect of the spatial distribution of late movers on the dynamics of the cooperative level. To reveal this, we consider a scenario where late movers are clustered within the lattice. Specifically, the cluster of late movers is designed to occupy a size of $N_L = L_C \times L_C$ grids. Thus, the fraction of late movers is $\rho_L = \frac{N_L}{N}$.

Compared to the scenario with randomly distributed late movers (see Fig. 5(a)), the structured cluster scenario exhibits a distinct pattern in cooperation dynamics (see Fig. 5(b)). The initial concentration of late movers in a cluster is conducive to an increased ultimate cooperative level. This analysis highlights the importance of spatial organization in promoting cooperation. The clustered late movers, by virtue of their proximity, can encourage the strategic choice of cooperation among late movers due to the probabilistic rule described in Eq. (3).

In contrast, the random distribution of late movers dissipates this influence, leading to a less favorable outcome for cooperation. These findings provide valuable insights into potential strategies for enhancing cooperation in real-world systems by promoting the spatial clustering of influential individuals or groups.

In Fig. 6, we broaden our exploration to comprehensively analyze how the size of clusters ($N_C = L_C \times L_C$) and the synergy factor (R) cooperatively impact the ultimate cooperative level within a system. We observe that, when the threshold is set at relatively low values (e.g., $\phi = 0$), there is a boundary line determined by smaller cluster sizes (N_L) of late movers and higher values of the synergy factor (R). Cooperation emerges only above this boundary line (see Fig. 6(a)).

As the threshold increases (e.g., $\phi = 3$), two boundary lines become noticeable. One boundary is enclosed by lower N_L and higher R, above which the ultimate cooperative level can approach 1. The other boundary is enclosed by lower N_L and lower R, below which the ultimate cooperative level can approach 0. It is noteworthy that under higher values of R, larger cluster sizes (N_L) can harm cooperation among the population (see Fig. 6(d,j,p)).

Finally, when the threshold reaches its maximum value ($\phi = 5$), the ultimate cooperative level remains at 1, irrespective of N_L and R (see Fig. 6(f,l,r)).

3.5.2. The extreme cases

In Fig. 7(a), where all early movers are defectors ($\rho_D = 0$), we see a distinct pattern based on the threshold value ϕ. When ϕ is low, the average cooperative level dramatically decreases and stabilizes at a lower value. Conversely, when $\phi = 5$, the cooperative level remains consistently at 1. This is because $n_C + n_L = \phi$ is satisfied for all groups, leading all late movers to choose cooperation according to Eq. (3). In Fig. 7(b), where all early movers are cooperators ($\rho_C = 0$), a different pattern emerges. When ϕ is high, the average cooperative level increases and stabilizes at a constant value. In contrast, when $\phi = 0$, the cooperative level is constantly 0. This is because $n_L \geq \phi$ is satisfied for

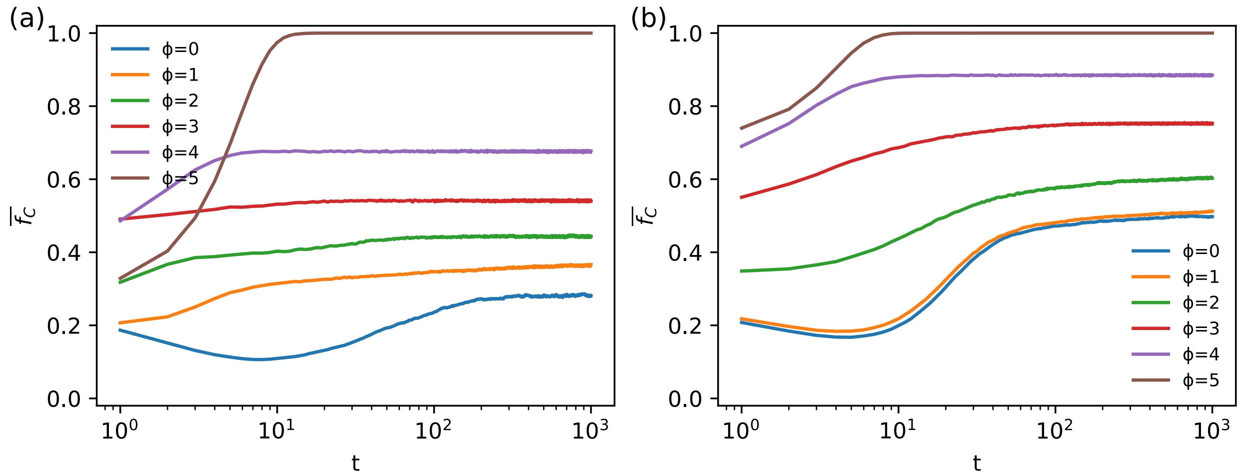

Fig. 5. Comparison of cooperation evolution with different distributions of late movers: (a) randomly distributed late movers and (b) clustered late movers with a cluster size of $N_L = L_C \times L_C = 40 \times 40$. Settings are as follows: $R = 5.2$, $\rho_L = 0.16$, and $\kappa = 0.5$. Each data point is the average of 50 independent simulations.

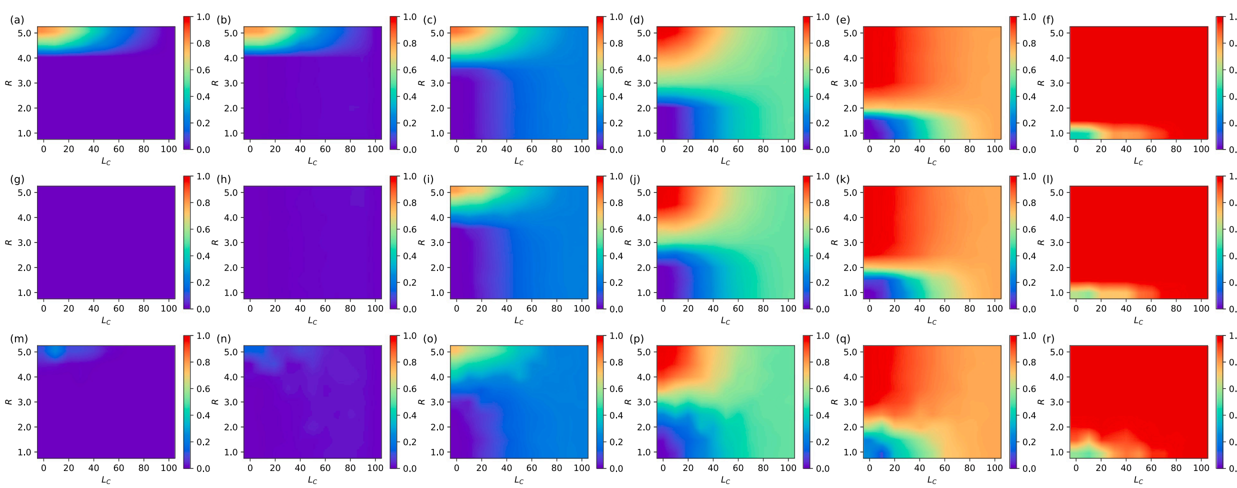

Fig. 6. The ultimate cooperative level in the $(L_C,\ R)$ plane with $\rho_C = \rho_D = (1-\rho_L)/2$. Left to right panels correspond to $\phi = \{0, 1, 2, 3, 4, 5\}$. Similarly, the panels from top to bottom showcase different κ values: $\kappa = 0.1$, $\kappa = 1$ and $\kappa = 10$.

all groups, prompting all late movers to choose defection according to Eq. (3). In Fig. 7(c), where the entire population consists of late movers ($\rho_L = 1$), the cooperation strategy choice is solely determined by ϕ, as per Eq. (3). Specifically, each late mover will choose to cooperate with probability $P(s_i = 1) = \exp\left(-\frac{n_L-(\phi-n_C)}{\phi-n_C}\right) = \exp\left(-\frac{5-\phi}{\phi}\right)$. As ϕ increases, the average cooperative level also increases and stabilizes at a higher value. Conversely, when $\phi = 0$, the cooperative level is consistently 0, as $n_L \geq \phi$ holds for all groups, leading all late movers to defect according to Eq. (3).

Let us next discuss how the fraction of late movers (ρ_L) and the synergy factor (R) jointly affect the ultimate cooperative level within the extreme cases. We can find, the results are robust for the extreme cases: $\rho_C = 0$ and $\rho_D = 0$. Figs. 8 and 9 can give us the images of the lowest and highest bound of the cooperative level in the system.

4. Conclusion

Our investigation into the evolutionary dynamics of cooperation in spatial threshold public goods games yields several noteworthy insights. Firstly, we find that the synergy factor – the multiplier applied to contributions before distribution – plays a pivotal role in shaping cooperative behavior. Higher synergy factors amplify the benefits of cooperation, incentivizing individuals to contribute to the public good and thereby increasing overall cooperative levels within the population. Conversely, lower synergy factors diminish these incentives, leading to reduced cooperation and potentially undermining collective welfare.

Secondly, the fraction of late movers emerges as a critical factor influencing cooperation dynamics. Late movers, by observing and adapting to the prevailing levels of cooperation in their vicinity, can significantly impact the overall cooperative strategy. Higher fractions of late movers generally enhance cooperation levels, as their adaptive behavior fosters the spread of cooperative strategies and counters the effects of defection.

Interestingly, the relationship between the fraction of late movers and cooperative outcomes is not linear. Optimal cooperation levels often arise when late movers constitute a moderate fraction of the population, balancing the benefits of strategic adaptation with the initial conditions set by early movers.

Thirdly, our study underscores the importance of the activation threshold in shaping cooperative dynamics. A higher threshold effectively screens out low-contributing individuals, fostering environments where cooperative strategies can flourish and mutual trust can be sustained. In contrast, lower thresholds may perpetuate free-riding behaviors, destabilizing cooperative efforts and hindering the establishment of sustainable collective action.

Moreover, our analysis of spatial distribution patterns reveals that clustering of late movers can significantly promote cooperation. When late movers are spatially concentrated, they are more likely to interact with cooperators and adopt cooperative strategies themselves,

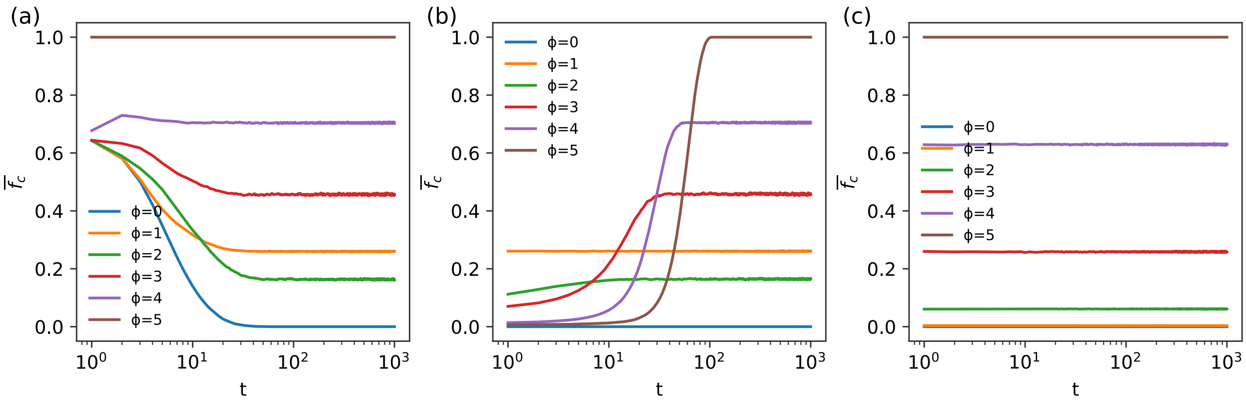

Fig. 7. The evolution of average cooperative level $\overline{f_C}$ with the special cases: (a) $\rho_D = 0$ and $\rho_L = 0.1$, (b) $\rho_C = 0$ and $\rho_L = 0.1$, (c) $\rho_L = 1$ for different threshold values. Settings: $R = 3.2$ and $\kappa = 0.5$.

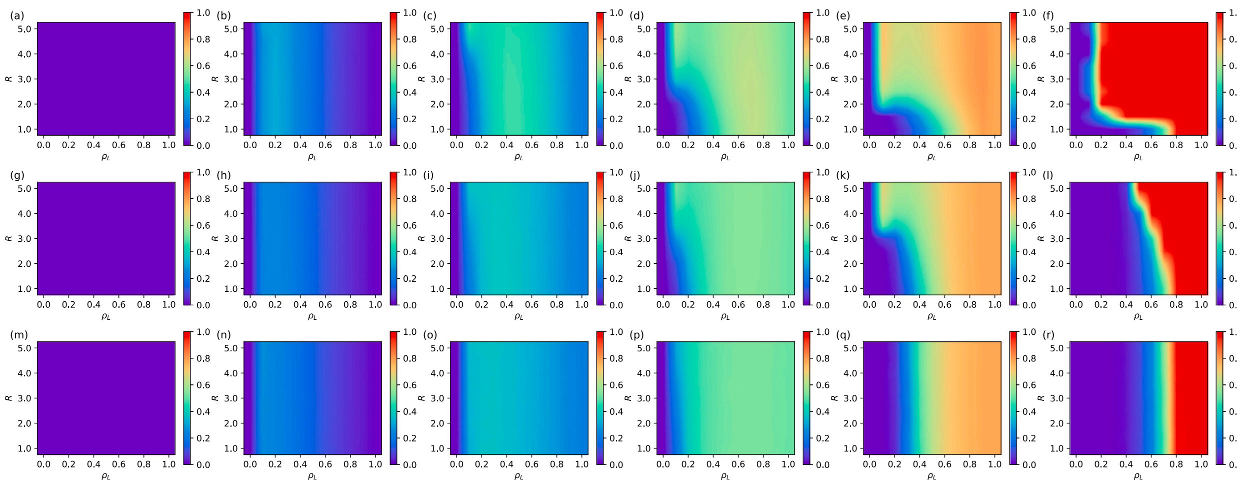

Fig. 8. The ultimate cooperative level in the (ρ_L, R) plane with $\rho_C = 0$, $\rho_D = 1 - \rho_L$. Left to right panels correspond to $\phi = \{0, 1, 2, 3, 4, 5\}$. Similarly, the panels from top to bottom showcase different κ values: $\kappa = 0.1$, $\kappa = 1$ and $\kappa = 10$.

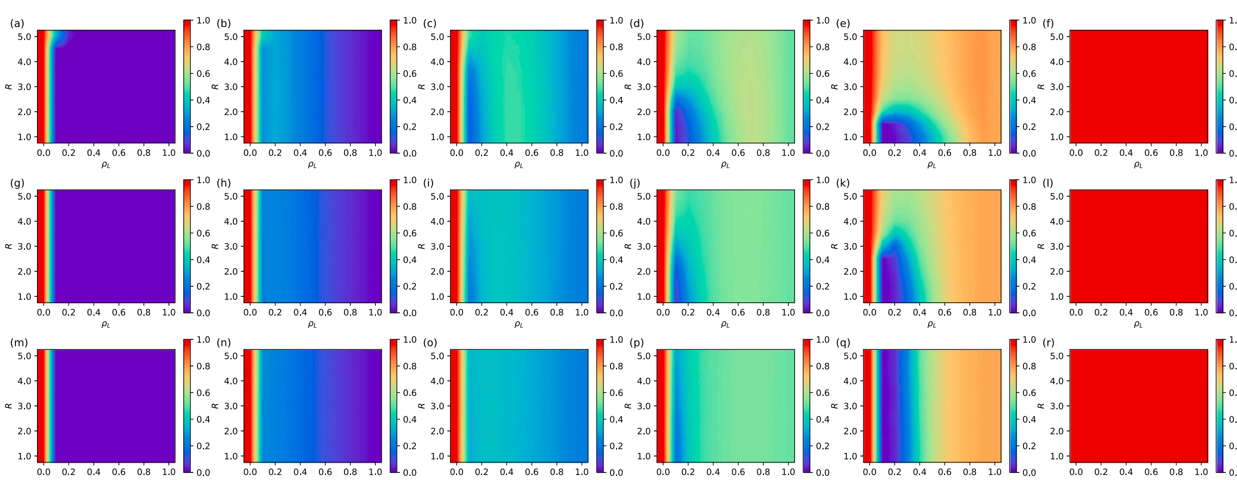

Fig. 9. The ultimate cooperative level in the (ρ_L, R) plane with $\rho_D = 0$, $\rho_C = 1 - \rho_L$. Left to right panels correspond to $\phi = \{0, 1, 2, 3, 4, 5\}$. Similarly, the panels from top to bottom showcase different κ values: $\kappa = 0.1$, $\kappa = 1$ and $\kappa = 10$.

leading to higher overall levels of cooperation compared to randomly distributed late movers.

The findings of this study open up several promising directions for future research. Firstly, extending the investigation to include other variants of public goods game models, such as those with variable threshold and synergy factor dynamics, could provide deeper insights into the mechanisms of cooperation. Secondly, examining the effects of various spatial configurations, such as small-world or scale-free networks, would help elucidate the role of interaction topology in shaping cooperative behaviors. Lastly, it would be particularly insightful to study the system's evolution under conditions where late movers and early movers can switch roles, thereby simulating a coevolutionary process [35], which could offer a richer understanding of the interplay between strategy and environment in cooperative dynamics.

The insights gained from this study have potential applications in real-world scenarios involving public goods and common resources. For instance, the results could inform strategies for managing environmental resources, public health initiatives, or social welfare distribution. Understanding how strategic late movers can enhance cooperation might help in designing policies or interventions that encourage individuals to contribute to the collective good, even in the presence of potential free-riders.

In conclusion, our study provides valuable insights into the intricate interplay between individual decision-making and collective cooperation dynamics in spatial threshold public goods games. By exploring diverse factors influencing cooperation, we contribute to a deeper understanding of how cooperative behaviors evolve and persist in complex social and ecological systems. These findings have practical implications across various domains, including environmental management, policy design, and community development, highlighting the importance of strategic adaptation and spatial organization in fostering sustainable cooperation.

CRediT authorship contribution statement

Zhehang Xu: Visualization, Validation, Methodology, Conceptualization. **Xu Liu:** Visualization, Validation, Software, Conceptualization. **Hainan Wang:** Validation, Software. **Longqing Cui:** Supervision, Methodology, Conceptualization. **Xiao-Pu Han:** Supervision, Methodology, Conceptualization. **Fanyuan Meng:** Writing – review & editing, Writing – original draft, Supervision, Methodology, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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