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Tunable resource allocation dynamics for interpreting economic complexity

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ABSTRACT

A lot of effort has been devoted to network structural interpretations of the economic complexity which can affect the level of economic growth and the activities of economic entities, although it remains unknown how network dynamical interpretations on which the economic complexity index operates affects its performance. Here, we regard export trade flows as the dynamics of resource allocation, and design a tunable resource allocation (TRA) process for interpreting economic complexity. The TRA could degrade into the classical economic complexity index (ECI) when the tunable parameter equals zero. We apply the TRA dynamics to the perfectly nested triangular matrix and the real country-product bipartite network from the world trade data to interpret economic complexity respectively. The parametric interpretations provide conceptual descriptions of the economic complexity index, shedding new light on a pattern of specialization of resource allocation, and the fluctuation of the economic complexity in countries and products. Moreover, the relationship of economic complexity between countries and products in the framework of TRA reveals that higher-ECI countries usually specialize in higher-ECI products while products not so as to countries, and the correlation strength varies with tunable parameters.

1. Introduction

Economic complexity aims to have a better understanding of the emergent complex behaviors in economic systems via the tools developed in complexity science [1–3]. The recent theoretical and empirical evidence has substantiated to show a strong link between the complexity of economies and their economic development [4]. Economic complexity seeks to take advantage of the potential value of economic database which can capture the complex structure of the data-driven network to condition the economic growth level and the country's productive structure [5]. Economic growth is usually accompanied by an upgrading of export products [6]. The product network space which is built by the country's database of export products could uncover the activities of economic entities. Generally, the time stamped world trade data of the economics were modeled by a country-product bipartite network with links representing export relations [7,8]. Motived by structural properties of product network space, two novel non-monetary economic complexity approaches were introduced to quantify the hidden growth potential of countries. One is called the Economic Complexity Index (ECI) which is introduced to infer information about countries' productive capabilities from product network space [9] and recently is proved to be a classic spectral clustering algorithm [10] mathematically. The other is named Fitness and Complexity Index (FCI) [3,11] through the iterative process like well-known PageRank [12] which evaluates both the quality and the quantity of links to a webpage in order to determine a relative score of that page's importance. However, it is not only the mathematics of the two approaches which are different, but also the obtained outcomes significantly diverge. Mariani et al. [13] and Liao et al. [14] gave a comparative analysis of the predictive abilities of economic complexity metrics. Sciarra et al. [15] combined the strengths of both methods and adopted a neat mathematical perspective based on linear-algebra tools within a bipartite-networks framework, which can still forecast countries' innovation potential and interpret the temporal dynamics of economic growth. In addition, the product network space exhibits a hierarchically nested structure which is already widely observed in economic systems originated from ecological systems [16–18]. Some researches focused on the nested structure of product network space, gave an alternative way to measure product complexity accordingly [19,20] or predicted the evolution of industrial ecosystems [21]. The key

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advantage of these approaches is that it captures the complex structure of the interactions between a large number of economic agents, which brings solutions to a variety of issues in observation, modeling, and prediction [22–24]. With the growing importance of services in global trade, economic development could appear through upgrading export products and services. Mishra et al. [25] incorporated the role of globalization of services to present a more complete picture of economic complexity across countries through a nonlinear iterative algorithm. The works developed data-driven approaches to the conceptual scheme for forecasting the evolution of a dynamic economic system, much like the network representation [8,26–29], the selective predictability scheme [11], and structural stability of physical dynamics [30], with the aim of removing details that are unimportant to the problem of interest.

Recently a lot of effort has been devoted to network structural interpretations of the economic complexity. For example, when applied to export data of country-product bipartite network, the economic (or say country) complexity index (ECI_c) and product complexity index (ECI_p) obtained by economic complexity approaches reveal a striking pattern of specialization across countries. Mealy et al. [10] considered that the ECI_c and ECI_p could also be interpreted as dimensionality reduction methods, which have close connections to diffusion maps. They found the relations that high-ECI_c countries (which tend to be richer) specialize in high-ECI_p products (which tend to be more technologically sophisticated), countries with low ECI_c (which tend to be poorer) specialize in low-ECI_p products (which tend to be less technologically sophisticated). Here, trade is regarded as a process of resource exchange and distribution, and we propose tunable resource allocation (TRA) to uncover economic complexity and find that the TRA is equivalent to the economic complexity index in a situation. A great deal of effort to analyze parametric interpretations provides conceptual descriptions of the economic complexity index, which shed new light on a pattern of specialization between the ECI_c and ECI_p, and the relationship of economic complexity between countries and products is not consistent, i.e. High-ECI_c countries specialize in high-ECI_p products, but products with High-ECI_p do not correspond to High-ECI_c. The rest of the paper is organized as follows. In section Materials and Methods, we introduce the architecture of a country-product bipartite network with links representing export relations. Then, the iterative resource allocation process is proposed, and the relationship between TRA and ECI is fully elaborated. In Section 3, the performance of TRA is studied when it is applied tothe perfectly nested triangular matrix and the country-product bipartite network. The effect of parameters in the TRA method is further analyzed in this section. Conclusions and discussions are drawn in the final section. And the potential relevance and applications of TRA are also discussed.

2. Materials and methods

2.1. The architecture of the country-product bipartite network

The world trade datasets of different products between countries by years are collected from publicly available databases. The data comprises 261 countries and 239 products (categories) from 1976 to 2014. We limit countries whose average GDP from 2006 to 2010 exceed 10^9 dollars, and the name and classification of products are according to the Standard International Trade Classification (SITC), 3-digits (for a more specific description, we can refer to the website https://unstats.un.org/unsd/classifications/Econ/). For the data analyzed in this article, we use the average data from 2006–2010, and finally obtain 238 products and 174 countries. Then we count the export volume of each country, i.e. v_{cp} which can be expressed as the country c exports the product p. It is necessary to check whether a trade interaction performs Revealed Comparative Advantage (RCA) [31]. We can estimate whether a country is a significant exporter of a product by calculating whether the share of a country's product in the world market is larger or smaller than the share of that product in the entire market. Mathematically,

$$RCA_{cp} = \frac{v_{cp}/\sum_p v_{cp}}{\sum_c v_{cp}/\sum_c \sum_p v_{cp}} \tag{1}$$

The natural cutoff $RCA \geq 1$ used to determine whether a trade interaction has Revealed Comparative Advantage (RCA). At this point the share of a country's product in the world market is equal or larger than the share of that product in the whole market. Therefore we can construct the binary country-product matrix a_{cp} based on the RCA matrix, where $a_{cp} = 1$ if $RCA > 1$, otherwise $a_{cp} = 0$. From the view of network science, we can represent the countries and products as two types of nodes, and $a_{cp} = 1$ means there is a link between country c and the product p. It is noted that there is no link between any two countries and two products. Consequently, a bipartite country-product network $G(C, P, L)$ is constructed by the matrix a_{cp}, where C and P represent the set of countries and products, respectively, and L is links which exist only between countries and products.

2.2. Tunable resource allocation

Most resources are ubiquitous in the nature, and how to allocate them is therefore of great importance in the sciences from both theoretical and practical perspectives. A variety of complex socioeconomic systems has been successfully described as networks whose interacting pairs of nodes are connected by links. To incorporate these interactions, a lot of work were proposed to model the socioeconomic system as an inter-agent resource competition network. To define the competition network, we shall generate a resource–agent system composed of N agents and R resources. A process of resource allocation can be simply described in terms of the interaction a_{ij} between agent i and j. Resources in each agent are allocated to its neighbors, and the allocation strategies are determined by the network topology structure. We design a TRA process for uncovering economic complexity. The basic idea is that trade is actually the process of resource exchange and distribution, thus we regard export trade flows as a process of resource allocation. The equation of resource allocation in a certain step of the resource allocation process is given by:

$$R_{j\to i} = \left(\frac{k_j^\lambda}{\sum_{u\in J(i)} k_u^\lambda} a_{ij}\right) I_j, \tag{2}$$

where $R_{j\to i}$ is the amount of resource allocation from j to i. I_j is the resources held by node j. $J(i)$ is the set of $i's$ neighbors. k_j is degree of the node j. The tunable parameter λ is used to nonlinearly adjust the weight of the degree.

Now, we apply the above rule of resource allocation to the country-product bipartite networks and obtain the final matrix of the resource allocation, i.e. $M_{cc'}$ and $M_{pp'}$. For the $M_{pp'}$, at first step, the initial resource value of each product is R_p, and we can set the initial value R_p as a constant like 1. The country obtains the resources from the network according to the equation of resource allocation,

$$R_c = \sum_{p=1}^{N} \left(\frac{k_p^\lambda}{\sum_{u\in J(c)} k_u^\lambda} a_{cp}\right) R_p. \tag{3}$$

At second step, we can feedback the resources to the product through the links between the product and countries. Again, each product obtains the resources from the countries according to the equation of resource allocation. A product becomes more complex if it gets more resources, thus we can regard the updated resource of the product as product complexity. The updated resource of the item $R_{p'}$ is represented by:

$$R_{p'} = \sum_{c=1}^{M} \left(\frac{k_c^\lambda}{\sum_{u\in J(p')} k_u^\lambda} a_{cp'}\right) R_c. \tag{4}$$

At the third step, we can copy this process to other products and then calculate final resource allocation matrix $M_{pp'}$. And we can do the similar procedure to get $M_{cc'}$.

2.3. The relationship between TRA and ECI

Using the tools developed in network science, diversity and universality are shown to be crude approximations of the variety of capabilities available in a country or required to export a product, respectively. Diversity is related to the number of products connected to a country. Ubiquity is related to the number of countries connected to a product. Thus, we can measure ubiquity or diversity simply by summing over the rows or columns of the matrix a_{cp}. Formally,

$$k_c = \sum_p a_{cp}, \tag{5}$$

$$k_p = \sum_c a_{cp}. \tag{6}$$

To generate a more accurate measure of the number of capabilities available in a country or required to export a product, Hartmann et al. [9] corrected the information that diversity and ubiquity carry by using each one to correct the other. For countries, this requires us to calculate the average ubiquity of its export products, and the average diversity of the countries that produce those products and so forth. For products, this requires us to calculate the average diversity of the countries that make them and the average ubiquity of the other products that these countries make. The matrix is used in the following equations.

$$M_{cc'} = diag(\frac{1}{k_c})a_{cp}diag(\frac{1}{k_p})a_{cp}^T, \tag{7}$$

$$M_{pp'} = diag(\frac{1}{k_p})a_{cp}^T diag(\frac{1}{k_c})a_{cp}. \tag{8}$$

The matrix $M_{cc'}$ connects country c and country c' according to the number of products that are exported by both. The matrix $M_{pp'}$ connects product p to product p'. The eigenvector associated with the second largest eigenvalue can capture the largest amount of variance in the system, and finally handle by Z-score. Hence, the ECI is defined as

$$ECI_c = \frac{v_c - \langle v_c \rangle}{\sigma(v_c)}, \tag{9}$$

$$ECI_p = \frac{v_p - \langle v_p \rangle}{\sigma(v_p)}. \tag{10}$$

where v_c and v_p are respectively the eigenvector of the matrix $M_{cc'}$ and $M_{pp'}$ associated with the second largest eigenvalue. $\langle\rangle$ stands for the average value, and $\sigma()$ represents the standard deviation.

The TRA degrades into the classical economic complexity index when the tunable parameter equals zero. Here, we explain the relationship between TRA and the above-mentioned ECI when $\lambda = 0$. As shown in Fig. 1abc, we give an example when the tunable parameter $\lambda = 0$, mark a product and the initial resource value is set as 1. At the first step, the country obtains resources from the product. The second step is a feedback of resources to the product, and we finally get the resource allocation matrix $M_{pp'}$ between products through the marked product according to the network and resource allocation rules. According to the Eqs. (3) and (4), we can get:

$$R_{p'} = \sum_{c=1}^{M} \frac{a_{cp'}}{k_{p'}} R_c = \sum_{c=1}^{M} \frac{a_{cp'} \sum_{p=1}^{N} \frac{a_{cp} R_p}{k_c}}{k_{p'}} = \sum_{p=1}^{N} R_p \frac{1}{k_{p'}} \sum_{c=1}^{M} \frac{a_{cp} a_{cp'}}{k_c}. \tag{11}$$

When we set the initial value R_p as unit.

$$R_{p'} = \sum_{p=1}^{N} \sum_{c=1}^{M} \frac{a_{cp} a_{cp'}}{k_{p'} k_c}. \tag{12}$$

The Eq. (12) can be expressed in terms of a matrix which is the same as the Eq. (8).

$$M_{pp'} = diag(\frac{1}{k_p})a_{cp}^T diag(\frac{1}{k_c})a_{cp}. \tag{13}$$

For the $M_{cc'}$, we can do the same procedure through the network structure and resource allocation. As shown in Fig. 1, we mark a country and give initial resources R_c. The resources are allocated to the commodity from the country in the network, and then the resource of country $R_{c'}$ is calculated. Finally $R_{c'}$ is given as following,

$$R_{c'} = \sum_{p=1}^{N} \frac{a_{c'p}}{k_{c'}} R_p = \sum_{p=1}^{N} \frac{a_{c'p} \sum_{c=1}^{M} \frac{a_{cp} R_c}{k_p}}{k_{c'}}. \tag{14}$$

If we set the initial value R_c as unit, we can obtain,

$$R_{c'} = \sum_{p=1}^{N} \sum_{c=1}^{M} \frac{a_{c'p} a_{cp}}{k_p k_{c'}}. \tag{15}$$

The Eq. (15) can be expressed in terms of a matrix which is the same as the Eq. (7).

$$M_{cc'} = diag(\frac{1}{k_c})a_{cp}diag(\frac{1}{k_p})a_{cp}^T. \tag{16}$$

Consequently, when the tunable parameter λ equals 0, the outputted matrix of resource allocation process is equivalent to ECI.

3. Results

3.1. TRA in the perfectly nested triangular matrix

Many recent researches have enlightened the nested pattern of ecological and economic systems [16–18]. The nested structure means that the network consists of generalist nodes and specialist nodes. The specialist nodes interact only with a small subset of generalist nodes, while the generalist nodes interact with (almost) all other nodes in the network. The property of nestedness is defined in terms of the organization of the nodes' neighborhoods (the set of nodes connected to a given one): for any two nodes, if the number of neighbors of the first is smaller than the second, then the neighborhoods of the former is contained in the latter. Taking the country-product bipartite matrix as an example, the rows and the columns of a country-product adjacency matrix are firstly swapped and rank-ordered by the sum of the presences in each of these rows and columns respectively. A nested triangular network in visualization is that the adjacency matrix shows a clear triangular pattern as revealed in Fig. 2a. The nested triangular feature has been enlightened in the branch of the economic complexity, which allows exploring the structure of economic systems via the tools developed in network science [9,11,32]. Thereupon, we use a perfectly nested triangular matrix to analyze TRA. The structure of the perfectly nested triangular matrix is shown in Fig. 2b, where the elements in each sequential row are a strict subset of those in the upper row and the elements in each sequential column are a strict subset of those in the upper column. Fig. 2c reveals the degree of the real country-product matrix and the perfectly nested triangular matrix. Although the degree of the real country-product matrix does not decrease linearly like the degree of the triangle, the degree decreases significantly as the node order increases. We can see that the generalist nodes and specialist nodes in the real country-product matrix could be distinguished.

Fig. 3 gives ECI values and the distribution of resource allocation in the perfectly nested triangular matrix with tunable parameters. It is obvious that $M_{cc'}$ and $M_{pp'}$ of the perfectly nested triangular matrix with TRA are the same, and normalizing the eigenvector corresponding to the second largest eigenvalue by Eqs. (9) and (10), we can get the ECI_c index and the ECI_p index which are also the same. Fig. 3a shows ECI values of the nodes in descending order of degree in the perfectly nested triangular matrix when the tunable parameter λ is −1, 0 and 1 respectively. When λ is 0, the index value is linear which is known as the original ECI. However, when parameters are adjusted to between −1 and 1 except 0, the index becomes nonlinear, especially the head and tail nodes are very sensitive to the parameters adjustment.

Moreover, Fig. 3bcd show a heat map of the $M_{cc'}$ matrix when the tunable parameter λ is −1, 0 and 1, respectively. The heat maps

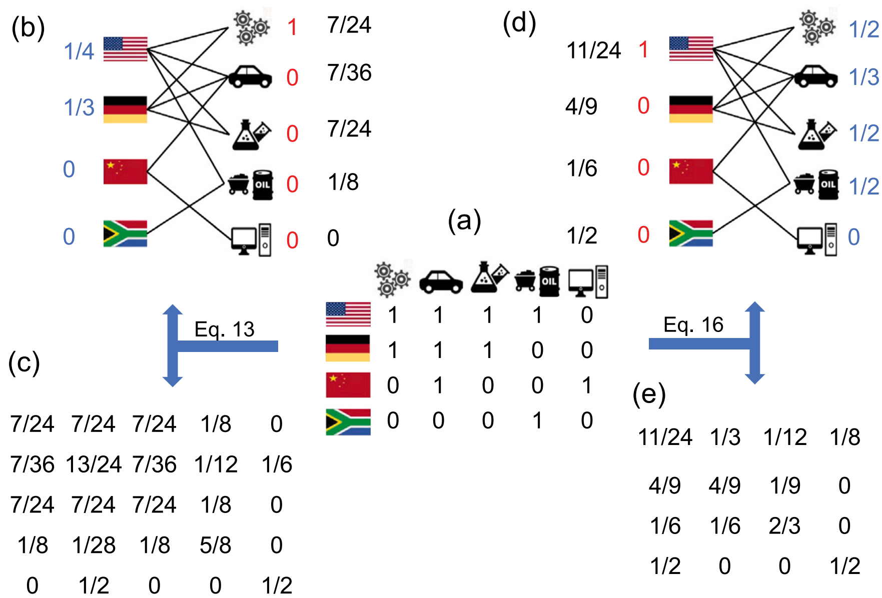

Fig. 1. An example of TRA process when parameter equals 0. (a) A simple matrix describes export trade. (b) The resource allocation when the tunable parameter $\lambda = 0$. We remark a product and the initial resource value is set as 1. (c) The resource allocation matrix $M_{pp'}$ when the tunable parameter $\lambda = 0$. Here we can use Eq. (13) to get $M_{cc'}$. (d) The resource allocation when the tunable parameter $\lambda = 0$. We remark a country and the initial resource value is set as 1. (e) The resource allocation matrix $M_{cc'}$ when the tunable parameter $\lambda = 0$. Here we can use Eq. (16) to get $M_{cc'}$.

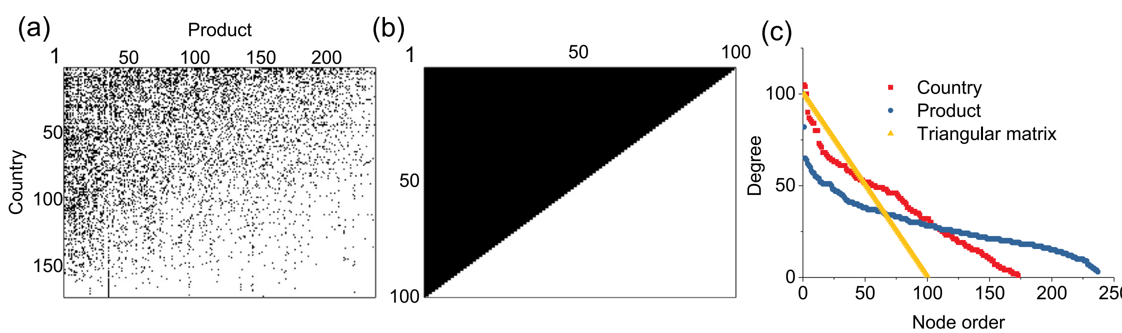

Fig. 2. The real country-product matrix and the perfectly nested triangular matrix. The rows and columns of these matrices are both reordered by descending degree (k). (a) The real country-product matrix contains 174 countries and 238 products. (b) The numbers of rows and columns in the perfectly nested triangular matrix are 100. (c) The degree of the real country-product matrix and the perfectly nested triangular matrix.

displayed by the three final resource allocation matrices are obviously different, which indicates that when extending to the case of non-linear, it results in a different distribution of economic complexity due to the different resource allocation matrix. From the view of the row, it displays that the node obtains the resources allocated by each column number node. We can observe that each node gets the most resources corresponding to its own when parameter equals −1. Then with the increase of the parameters, the obtained resources become less. When the parameter is 0, the resource obtained by the node is mainly positively related to its own ECI. And when the parameter is 1, the resource obtained by the node is mainly related to the ECI values of its neighbors. Then we look at the results from the columns, it displays that the node allocates the resources to each row node in this column. We can observe that each node allocates the most resources corresponding to its own when parameter equals −1, which becomes weak as the parameter increases.

Overall, when the parameter is −1, we could see that the resource allocation is homogenized from the heat map of the resource allocation of matrix $M_{cc'}$. When the parameter equals 0, the resource distribution tilts to the nodes with the large degree. According to the resource distribution formula, the distribution generates linear distribution according to its own degree, so that we can observe a gradual change. However, when the parameter reaches 1, it is clear from the resource allocation formula that the allocation based on the degree of neighbors is nonlinear, thus what we see is an obvious layering phenomenon.

3.2. TRA of the economic complexity analysis in country-product matrix

We first analyze the TRA of economic complexity for countries in country-product matrix. The binary country-product matrix is constructed by the world trade data which is from 2006 to 2010 on a 5-year average, and then processing data by RCA. According to the TRA process, we can calculate the $M_{cc'}$, and obtain the eigenvector corresponding to the second largest eigenvalue. We handle the eigenvector by z-score to get the country's ECI value. We focus on the Group of Twenty (we say G20, but here we focus on 19 individual countries except European Union) which represents around 80% of the world's economic output, two-thirds of global population and three-quarters of international trade. Fig. 4a shows the ranking of countries in the G20 under different parameters. We can see that the rankings of the countries JPN, GBR and SAU remain stable when the parameter λ changes from −1 to 1, the rankings of the countries DEU, FRA, ITA, MEX and RUS increase, and the rankings of some countries decrease, such as USA and KOR. We also find that the top 20 countries have always been relatively stable, but the ranking of countries after the top 20 fluctuates with the parameters. Of course, some of these countries

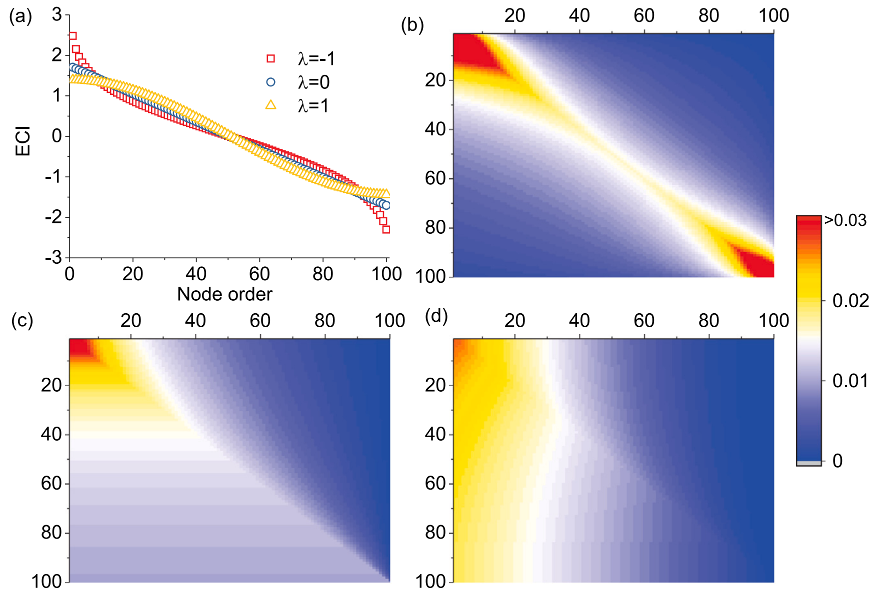

Fig. 3. TRA in the perfectly nested triangular matrix. (a) Three parameters −1, 0, 1 corresponds to ECI values. (b,c,d) The heatmap of the resource allocation matrix when the parameter is −1, 0, 1. It is noted that starting from the row, heatmap means that the node of the row number obtains the resources allocated by each node in this row. From the column, heatmap means that the node of the resources of the node of column number are allocated to each node in this column.

ranking after 20 have always been declining and also rising, but except for SAU which has been staying around 170. In addition, The heat maps of Fig. 4bcd illustrate the relationship between G20 in the resource allocation matrix $M_{cc'}$ when the parameters are −1, 0 and 1, respectively. From the view of row, each country in the G20 gets the resources given by the G20 countries, and we can observe that each country gets the most resources corresponding to its own. With the increase of the parameters, the obtained resources are more and more, but JPN and GBR are getting less. Then from the view of column, the results show the allocation of resources by these countries. We can find that as the parameter increases, the higher-ranking countries, such as DEU, USA, FRA, ITA, CHN, TUR, BRA, IND and ZAF, will enable the G20 countries to obtain more resources after allocating resources. However some countries are exceptions, such as JPN and GBR, and some countries have little change, such as MEX, IDN, ARG, CAN, AUS, RUS and SAU. Therefore, different parameters which will change $M_{cc'}$ will result in different national ECI values. An important potential application here is to use different parameters to observe the changes of national ECI and find differences in the allocation of national resources, thereby we can promote the country's ECI, or explore the country's potential.

We then analyze the TRA of economic complexity for products in country-product matrix. Input the binary country-product matrix and according the TRA process, we can calculate the $M_{pp'}$ and the eigenvector corresponding to the second largest eigenvalue of the resource allocation matrix $M_{pp'}$. After performing z-score process, we can get the ECI_p of product. We sort the ECI_p, and focus on the ranking of the top 10 products (namely Top10) when $\lambda = 0$ for analysis. Fig. 5a shows the ranking of Top10 under different parameters. We can find the ECI_p is susceptible to large fluctuations by the parameters when processed using the TRA method, especially when the parameter around 0. The ranking changes quite a lot compared to the ranking of countries which is relatively stable. For example, when the parameter changes from −1 to 1, there is a downward trend in the ranking of products numbered 2 (Parts, nes of and accessories for machines of headings 751 or 752), products numbered 4 (Gramophones, dictating machines and other sound recorders) and products numbered 6 (Thermionic, microcircuits, transistors, valves, etc.). Apart from that, the ranking of some products rises first and then falls to 9; some products keep rising and then stay around 10. Therefore, through TRA, we can conclude that the economic complexity of the product is very sensitive to parameters. In addition, we also study the relationship between Top10 in $M_{pp'}$ when the parameters are −1, 0 and 1, respectively, as shown in the heatmaps of Fig. 5bcd. From the view of row, the Top10 products get the resources given by the Top10, and each product gets the most resources corresponding to its own. Then, as the parameters increase, the obtained resources become less and less, which is the opposite of $M_{cc'}$. Then we look at the column, that is, the result of these products after resource allocation. We can find that after these commodities allocate their own resources, the resources obtained by the commodities eventually decrease with the increase of the parameter, which is also different from the G20 matrix in $M_{cc'}$. Therefore, the $M_{pp'}$ will change for different parameters, resulting in different ECI_p of the final product. In fact, there is a very important potential application that using different parameters to fine-tune and observe the complexity of the product so as to grasp the development of technology, because the complexity of the product has a complicated relationship with the market and technological innovation.

3.3. The economic complexity relation between countries and products

Mealy et al. [10] noted that the ECI_c and ECI_p can also be interpreted as dimensionality reduction methods which have close connections to diffusion maps, and gave the relations between ECI_c and ECI_p. High-ECI_c countries (which tend to be richer) specialize in high-ECI_p products (which tend to be more technologically sophisticated), and countries with low ECI_c (which tend to be poorer) specialize in low-ECI_p products (which tend to be less technologically sophisticated). However, these findings are obtained when the tunable parameter equals 0, that is to say, in the linear case. In order to extend to the nonlinear, we learn about the relationship of economic complexity between countries and products according to the TRA. We first count the average ECI_p of goods exported by a country in Fig. 6abc. The result shows that High-ECI_c countries specialize in high-ECI_p products, and countries with low ECI_c specialize in low-ECI_p products. When the parameter equals to 0, the description seems very significant, but when the parameters are −1 and 1, the result is not so significant.

Next, as shown in Fig. 6def, we in turn count the average ECI_c of the countries exported to one product, and the relationship that high-ECI_p products specialize in high-ECI_c countries, and low-ECI_p

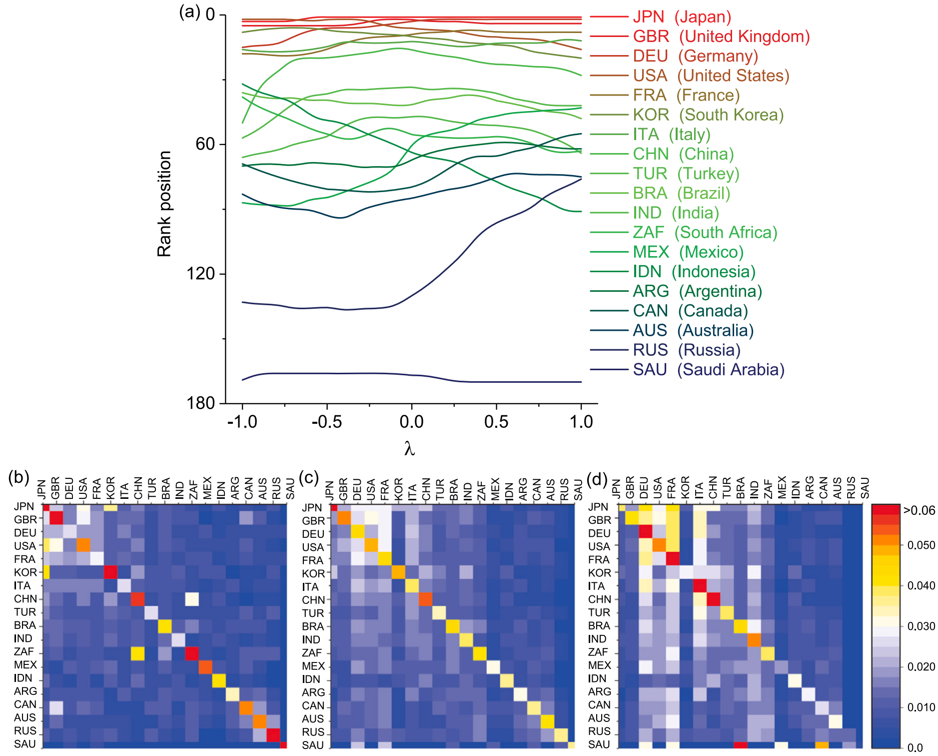

Fig. 4. TRA of ECI_c in the country-product matrix. (a) the ranking of G20 (except EU) when parameter is adjusted from −1 to 1. (b,c,d) The heatmap of the resource allocation matrix $M_{cc'}$ with tunable parameters (−1,0,1). $M_{cc'}$ could reveals resource allocation among countries in the country-product matrix.

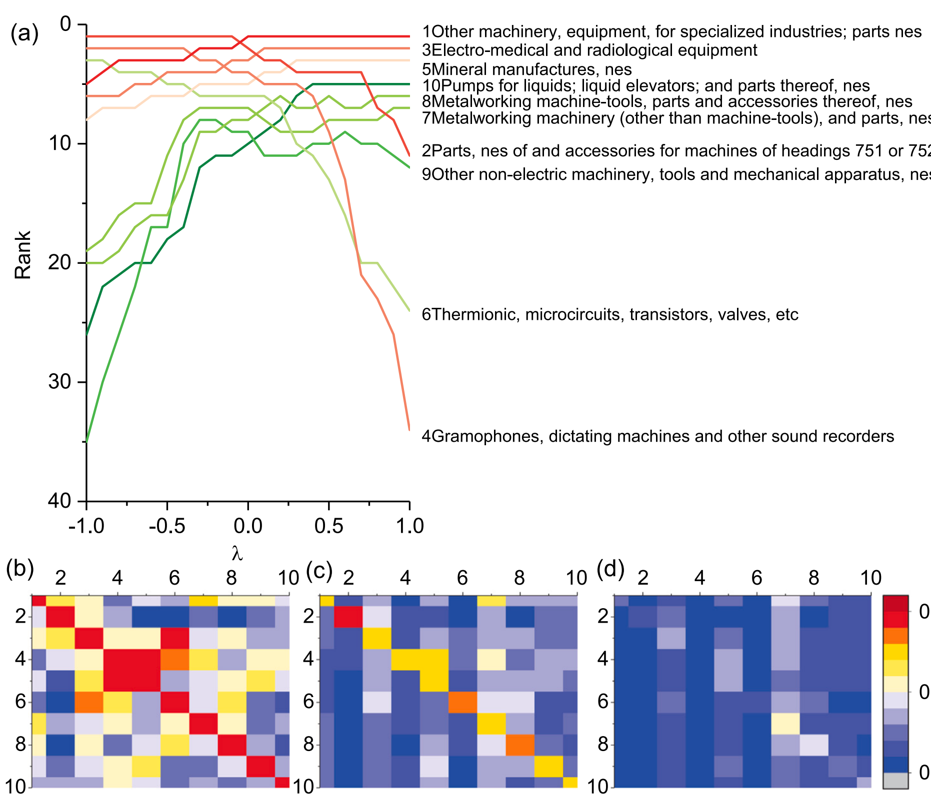

Fig. 5. TRA of ECI_p in country-product matrix. (a) The ranking of Top10 products when parameter is adjusted from −1 to 1. It is noted that product number is ranking position when parameter equals 0, the product name is formally from SITC. (b,c,d) The heatmap of the resource allocation matrix $M_{pp'}$ with tunable parameters (−1,0,1). $M_{pp'}$ could reveals resource allocation among products in the country-product matrix.

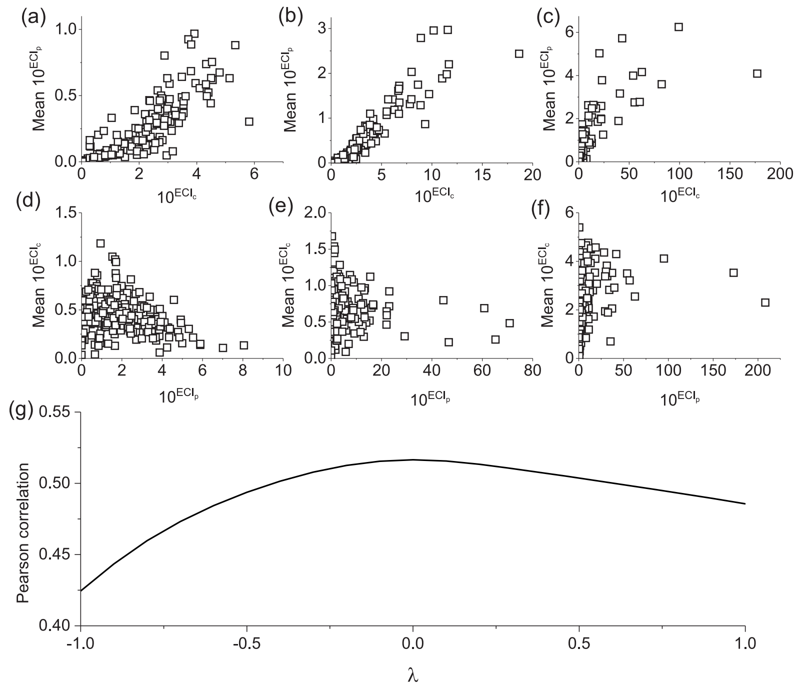

Fig. 6. The economic complexity relation between ECI_c and ECI_p. (a,b,c) The relation between 10^{ECI_c} of the country and mean 10^{ECI_p} of the products which the country exports when the parameter is −1, 0, 1 respectively. (d,e,f) The relation between 10^{ECI_p} of the product and mean 10^{ECI_c} of the countries which the product connects when the parameter is −1, 0, 1 respectively. (g) Pearson Correlation between ECI_c and ECI_p corresponding to the values of country and product of link list in country-product networks.

products specialize in low-ECI_c countries is not significant; for this reason, we give the Pearson Correlation between ECI_c and ECI_p in Fig. 6g. The country-product link list corresponds to the ECI_c and ECI_p values, and then we find the Pearson Correlation of this list to analyze the economic complexity relation between countries and products. It is found that with the parameter changes from −1 to 1, the correlation rises first, reaches the maximum value near 0.55 when the parameter is 0, and then starts to decrease. It indicates that the economic complexity relation between countries and products is not particularly significant, but moderate.

4. Conclusions and discussions

Substantial works has substantiated to develop data-driven approaches into the conceptual scheme of forecasting the evolution of a dynamical economic system as in the case of physical dynamics which aims to remove details that are not important for the problem of interest. Although substantial network structural interpretations aim to uncover the economic complexity, it remains unknown how the tunable parameters on which the economic complexity index operates affects its performance. As a remedy, we designed a TRA process for interpreting economic complexity. The TRA is equivalent to the Economic Complexity Index (ECI) when the tunable parameter equals zero. We explored the TRA of economic complexity in the perfectly nested triangular matrix and the real country-product matrix, respectively, and took G20 group and the most complex products as example to explore the effect of resource allocation with tunable parameters. We studied distribution of resource allocation in the perfectly nested triangular matrix and the real country-product matrix with tunable parameters, respectively, and also investigated the fluctuations of the ranking changes of G20 group and the most complex products under TRA. In addition, we extended the analysis of correlation between countries complexity and products complexity from the linear to nonlinear and discussed the parametric interpretations providing conceptual descriptions of the economic complexity index, which also shed new light on a pattern of specialization of the complexity between country and product. The results revealed that the relationship of economic complexity between countries and products is not consistent, i.e. High-ECI countries specialize in high-ECI products, and countries with low-ECI specialize in low-ECI products, but high-ECI products are not to high-ECI countries, and low-ECI products are not to low-ECI countries. It is interesting that when the tunable parameter equals zero, the Pearson relationship between ECI_c and ECI_p reaches highest value.

Our work could bridge resource allocation dynamics and economic complexity to uncover the parametric interpretations in the data-driven network. An important potential application here is to use different parameters to observe the fluctuations of national fitness and penetrate differences in the allocation of national resources, thereby we can capture the predictability of economic growth and uncover the dynamism of economic entities. More importantly, understanding the complexity of international trade is critical for a variety of issues ranging from quantifying the competitiveness of individual nations to forecasting the collective evolution of the world economy. The tunable economic complexity via resource allocation dynamics could support a solution for disentangling the perturbations and instabilities that burst out in complex socioeconomic systems such as financial crises, pandemics or social events. In addition, we could extend parametric interpretations by the TRA dynamics to explore the complexity of the product which has a complicated relationship with the market and technological innovation, and this model would offer a useful reference for the government to undertake industry strategies. These open questions ask for future study.

CRediT authorship contribution statement

Ren Zhuo-Ming: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Validation, Writing – original draft, Writing – review & editing. **Zhao Li:** Formal analysis, Methodology, Writing – original draft, Writing – review & editing. **Du Wen-Li:** Formal analysis, Methodology, Writing – original draft, Writing – review & editing. **Weng Tong-Feng:** Formal analysis, Writing – original draft, Writing – review & editing, Supervision . **Liu Chuang:** Methodology, Writing – original draft, Writing – review & editing. **Kong Yi-Xiu:** Methodology, Writing – original draft, Writing – review &

editing. **Zhang Yi-Cheng:** Conceptualization, Formal analysis, Writing – original draft, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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